Complex Number:

A number of the form $z = x + iy$ where $x, y \in R$ and $i = \sqrt{-1}$ is imaginary number is called complex number.

A complex number may also be defined as an order pair $z = (x, y)$ of real number.

$z = x + iy = (x, y)$

Here,

$i = \sqrt{-1}$ is imaginary number. Also $i^2 = -1, i^3 = i^2.i = -i, i^4 = i^2.i^2 = (-1).(-1) = 1$

- x is called real part of $z = Re(z)$
- y is called imaginary part of $z = Im(z)$

So, $z = x + iy = (x, y) = z = Re(z) + Im(z)$

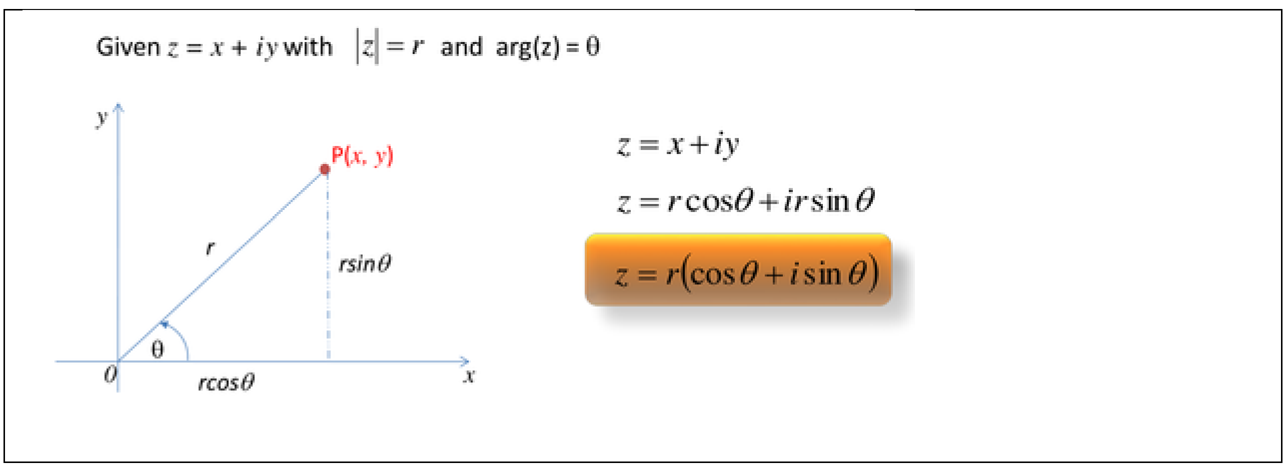
- If $x \neq 0, y = 0$ then $z = x$ is purely real
- If $x = 0, y \neq 0$ then $z = x$ is purely imaginary

Conjugate of complex number: The complex conjugate, or simply the conjugate, of a complex number $(x - iy)$ and denoted by $\bar{z}$ that is $\bar{z} = x - iy$.

The number $\bar{z}$ may also be defined as an order pair $z = (x, -y)$ of real number.

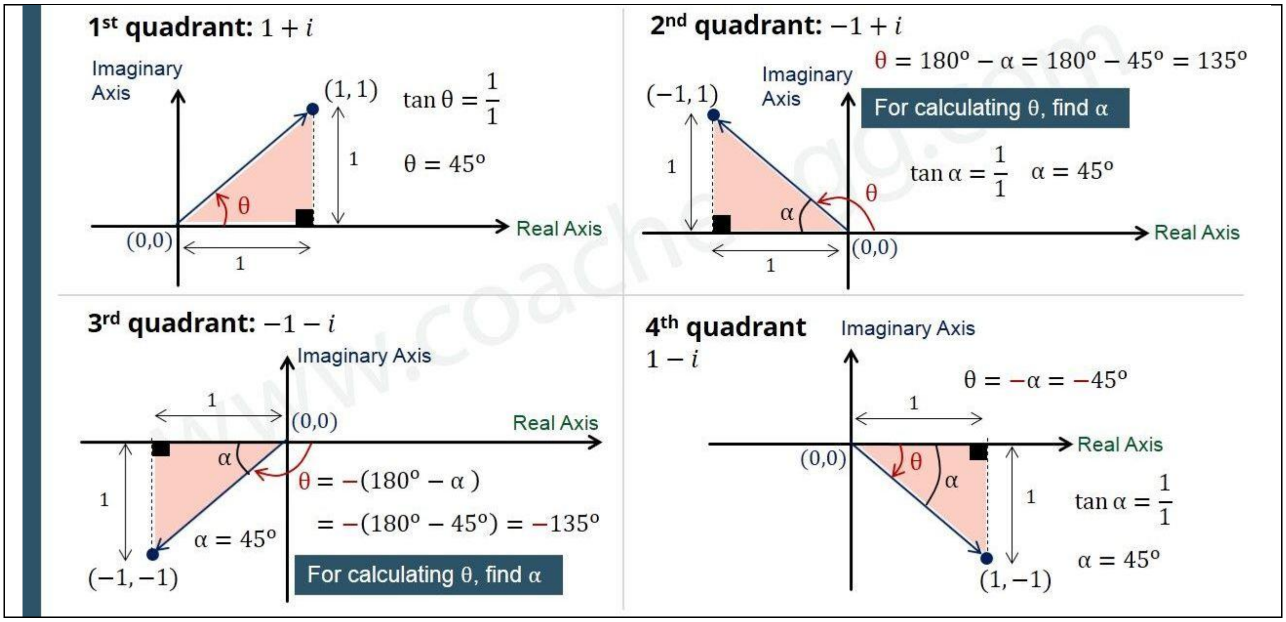
Modulus The modulus, or absolute value, of a complex number $z = x + iy$ is defined as the nonnegative real number $\sqrt{x^2 + y^2}$ and denoted by $|z|$ that is $|z| = \sqrt{x^2 + y^2}$

Argument: The argument of z is the angle between the positive real axis and the line joining the point to the origin. It is denoted by "θ" or "φ". It is measured in standard units "radians".



In this diagram, the complex number is denoted by the point P. The length OP is known as magnitude or modulus of the number, while the angle at which OP is inclined from the positive real axis is said to be the argument of the point P.

The argument complex number $z = x + iy$ is defined by $\theta = \tan^{-1}(\frac{y}{x})$



Write down the modulus and argument of the complex number i) $z = 3 + 5i$ ii) $z = 3 - i$	iii) $z = 4 - 4i$ iv) $z = 3 + 5i$ v) $z = 5i$ vi) $z = -1 - 3i$

Polar form of complex number:

The polar form of a complex number $z = x + iy$ is defined by $z = r(\cos\theta + i\sin\theta)$ where $|z| = \sqrt{x^2 + y^2}$, $x = r\cos\theta$, $y = r\sin\theta$ and $\theta = \tan^{-1}(\frac{y}{x})$

Transform to polar form:

Example-1: $z = 4 + 4i$

Here, $|z| = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ and $\theta = \tan^{-1}(\frac{4}{4}) = \tan^{-1}(1) = \frac{\pi}{4}$

Then polar form is $z = r(\cos\theta + i\sin\theta) = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Transform to rectangular/Cartesian form:

$z = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

here $r = 4\sqrt{2}$ and $\theta = \frac{\pi}{4}$ then

We have $x = r\cos\theta = 4\sqrt{2}\ \cos\frac{\pi}{4} = 4\sqrt{2}\ \frac{1}{\sqrt{2}} = 4$ and $y = r\sin\theta = 4\sqrt{2}\ \sin\frac{\pi}{4} = 4\sqrt{2}\ \frac{1}{\sqrt{2}} = 4$

So $z = x + iy = 4 + 4i$

The following problems Transform to polar form

(i) $z = -2 + i$	**(vi)** $z = 3 + 3i$
(ii) $z = 4 - i\sqrt{2}$	**(vii)** $z = -2 + 4i$
(iii) $z = 2 - 2i$	**(viii)** $z = 1 + i$
(iv) $z = -1 - i\sqrt{3}$	**(ix)** $-3i$
(v) $z = 4 + 5i$	**(x)** $-1 - 3i$
	(xi) $1 - \sqrt{3}i$

(xii)

Example 4. Express the following complex numbers in polar form.

(i) $2 + i\,2\sqrt{3}$ (ii) $-3 + 3i$ (iii) $-\sqrt{6} - i\sqrt{2}$ (iv) $-5i$

(xiii)

Represent the following complex numbers in polar form.

i. $z = 2 + 2j$ ii. $z = 5 - 12j$

Exercise 1.8

State the following complex numbers in polar form.

i. $z = 3 - j$ ii. $z = 9 + 3j$

Example 1.12

Express the following in $z = a + bj$ form.

i. $z = 6(\cos 60^o + j\sin 60^o)$ ii. $z = \sqrt{2}(\cos 135^o + j\sin 135^o)$

Exercise 1.9

i. $z = 8(\cos 90^o + j\sin 90^o)$ ii. $z = \sqrt{3}(\cos 75^o + j\sin 75^o)$